Design of Low-Pass Elliptic Filters by Means of Cascaded Microstrip Rectangular Elements

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Abstract —**A new method for synthesizing nonredundant low-pass elliptic filters in a microstrip configuration is presented. The realization consists of the cascade connection of proper rectangular elements, each one corresponding to four reactive elements of the lumped-constant prototype. This allows an effective control of parasitics and unwanted reactances which results in the possibility of realizing higher order filters with cutoff frequencies up to *X*-band. Fifth- and seventh-order filters were fabricated on alumina substrates showing very good performance, particularly in the passband.**

I. Introduction

THE NEEDS FOR reduced size and weight and for low fabrication cost render a printed circuit configuration particularly favorable for a number of microwave components. Low-pass filters with elliptic-function response lend themselves to the printed circuit version. They allow, in fact, very high selectivities to be achieved with a reasonable number of elements offering a substantial size advantage over other low-pass filter forms.

Synthesis techniques of distributed elliptic filters using commensurate line sections require the insertion of redundant lengths of transmission line [1]–[3]; the increased number of elements limits the above mentioned advantages and may cause unacceptable insertion loss in the passband, particularly when a microstrip version is used.

On the other hand, the realization of printed version microwave filters based on nonredundant design techniques has been limited to stripline configurations and with cutoff frequencies up to a few gigahertz. Such techniques are based on the translation of each lumped element into a low or high impedance line section. The presence of unwanted reactances [4] and of parasitics due to discontinuity effects [5] make necessary adjusting or optimization procedures which render the synthesis rather involved.

A nonredundant realization of third-order low-pass elliptic filters as microstrip rectangular elements has been presented in recent years [6]. The synthesis procedure was based on an equivalence between a single microstrip element and the entire lumped cell of the starting prototype. This led to the reduction of the number of distributed elements and of the discontinuities, so minimizing the inconveniences of the other nonredundant techniques and allowing the design of filters with higher cutoff frequencies. The inability of cascading those microstrip elements, however, prevented the synthesis of higher order filters.

By introducing another type of microstrip rectangular element, the possibility of synthesizing high-order low-pass elliptic filters is demonstrated in the present paper. The realization consists of the cascade connection of rectangular microstrip elements, each one corresponding to four reactive elements of the lumped prototype. Very high passband performances of the filters are so obtained because of the better approximation used with respect to semilumped techniques. Experimental results of fifth- and seventh-order filters with 8-GHz cutoff frequencies are presented which confirm the theoretical expectations. It is finally shown that passband attenuation even lower than that of the prototype can be achieved through a simple adjusting procedure for compensating the reactances due to higher order resonant modes.

II. Elementary Rectangular Structures

Fig. 1 shows the geometry of the two types of planar rectangular structures which can be used for realizing nth order (n odd) microstrip elliptic low-pass filters. With reference to the sides where the ports are located, we shall refer to the structure of Fig. 1(a) as the parallel (P) element and to that of Fig. 1(b) as the normal (N) element. P-elements have been shown to be suitable for the realization of third-order filters [6]; they have the basic limitation of not allowing the construction of higher order filters by the simple cascade of such elements. This limitation is overcome by the use of N-type elements which allow more complex structures to be built through the direct connection of both types of elements; in particular, port A of a N-element can be connected to port B of another N-element as well as to any port of a P-element.

By expanding the electromagnetic field inside a rectangular element in terms of $TM^{(z)}$ resonant modes, the impedance matrix can be expressed in the form of a partial fraction expansion [7]. By retaining only the first two terms, corresponding to the zero frequency $TM^{(z)}{}_{00}$ resonant mode and to the $TM^{(z)}{}_{10}$ resonant mode, a good approximation is obtained in the lower frequency range [8].

Under such an approximation the impedance parameters

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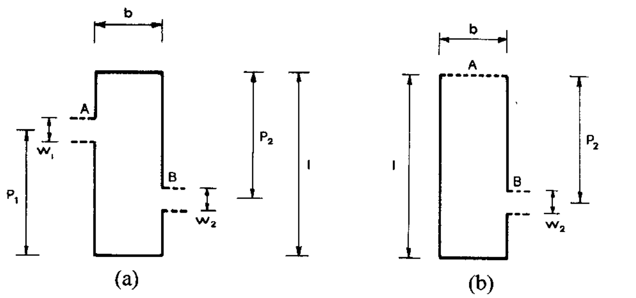

Fig. 1. Geometry of the (a) parallel-type and (b) normal-type microstrip elements.

of the structures of Fig. 1 reduce to

$$Z_{mn} = h_0\left(\frac{1}{j\omega} + \frac{2j\omega q_m q_n}{\omega_{10}^2 - \omega^2}\right), \qquad m, n = 1, 2 \tag{1}$$

where, for the P-type

$$h_0 = \frac{t}{\epsilon b l}$$

$$q_i = (-1)^i \cos\frac{\pi p_i}{l}\, \mathrm{Sn}\left(\frac{\pi w_i}{2l}\right)$$

$$\mathrm{Sn}(x) = \frac{\sin x}{x} \qquad \omega_{10}^2 = \pi^2/(\mu\epsilon l^2). \tag{2}$$

For the N-type, the same expressions ((1) and (2)) hold, with the exception of the q_1 coefficient which is equal to unity

$$q_1 = 1. \tag{3}$$

In (2), t, μ, and ϵ are the substrate's thickness, permeability, and permittivity, respectively; as quoted in Fig. 1, b, l are the dimensions of the rectangular element, p_1, p_2 the positions of the ports, and w_1, w_2 their widths.

As is well known, one of the major problems both in the analysis and the design of microstrip structures consists of accounting for fringing field effects. Accurate results in wide frequency ranges are obtainable by the technique illustrated in [7]; this technique consists of ascribing to the structure effective dimensions and an effective dynamic permittivity depending on the field distribution of the resonant mode, as done by [9], for calculating the resonant frequencies of microstrip resonators. This technique has been adopted in [6] for synthesizing third-order low-pass elliptic filters as P-type microstrip elements.

In the limited frequency range where the two mode approximation is valid, however, good results can be obtained using a unique effective model of the microstrip structure; though more approximate, particularly in wide frequency ranges, this approach permits a notable simplification of the synthesis procedure, as will be shown later.

We shall then assume that b, l, p_1, w_1, p_2, w_2, and ϵ in (2) are the effective parameters of the rectangular elements, which can be evaluated, for instance, through the formula quoted in [10]. In particular, for the purpose of calculating the effective permittivity ϵ and the effective width b, the rectangular element may be regarded as a microstrip line section.

This has suggested the possibility of extending the present synthesis technique to the design of filters employing noncommensurate line sections and stubs [13], which preserves the advantage of reducing impedance discontinuities[1].

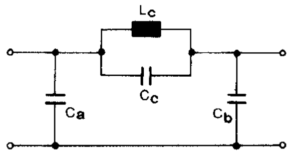

Fig. 2. Equivalent circuit of the microstrip rectangular elements of Fig. 1.

It can be easily seen that (1) leads to the equivalent circuit shown in Fig. 2, which has the basic structure of a third-order elliptic filter. The values of the components are related to the parameters of the equivalent circuit

$$C_a = \frac{C_0}{1 - q_1/q_2} \qquad C_b = \frac{C_0}{1 - q_2/q_1}$$

$$C_c = \frac{C_0}{2}\frac{1 + 2q_1q_2}{(q_1 - q_2)^2} \qquad L_c = \frac{2\mu t}{\pi^2}(q_1 - q_2)^2 l/b \tag{4}$$

where

$$C_0 = \frac{1}{h_0} = \frac{\epsilon b l}{t}. \tag{5}$$

In the case of the P-element the values of C_a, C_b, C_c, L_b can be controlled independently by varying the dimensions l, b of the structure and the position of the ports p_1, p_2; on the contrary, in the case of the N-element, for which the position of the first port is fixed, the capacitors of the equivalent circuit are related, because of (3), by

$$C_a = \frac{C_b}{2}\frac{C_b - 2C_c}{C_b + C_c}. \tag{6}$$

Equation (4) shows that, as can be expected, the product $l \cdot b$, thus the area of the element, affects the values of the capacitors, while the inductance L_c depends on the shape ratio l/b. Depending on q_1 q_2, thus on the position of the ports, the capacitors may also assume negative values, while L_c is always positive. For a fixed location $p_1 > l/2$ of the first port of a P-element, Fig. 3(a) shows the behavior of C_a/C_0, C_b/C_0, C_c/C_0 as functions of the position of the second port. As can be seen, positive values of all capacitors are obtained only if the two ports are at opposite sides with respect to $l/2$ and that only one capacitor at a time may be negative. In the case of the N-element, illustrated in Fig. 3(b), the capacitor C_b cannot assume negative values. In both cases, a wide range of variability of the equivalent parameters is obtainable. This property makes it possible in most cases to synthesize a lumped circuit as that of Fig. 2 as a microstrip element of P- or N-type.

[1]A synthesis technique based on a transmission line approximation model would result not fully adequate when applied to microstrip configuration. The finite width of the microstrip line sections, which gives place to the sin x/x term in (2), is not accounted for in a monodimensional approach, while this term cannot be neglected in a correct synthesis procedure, particularly in high frequency range.

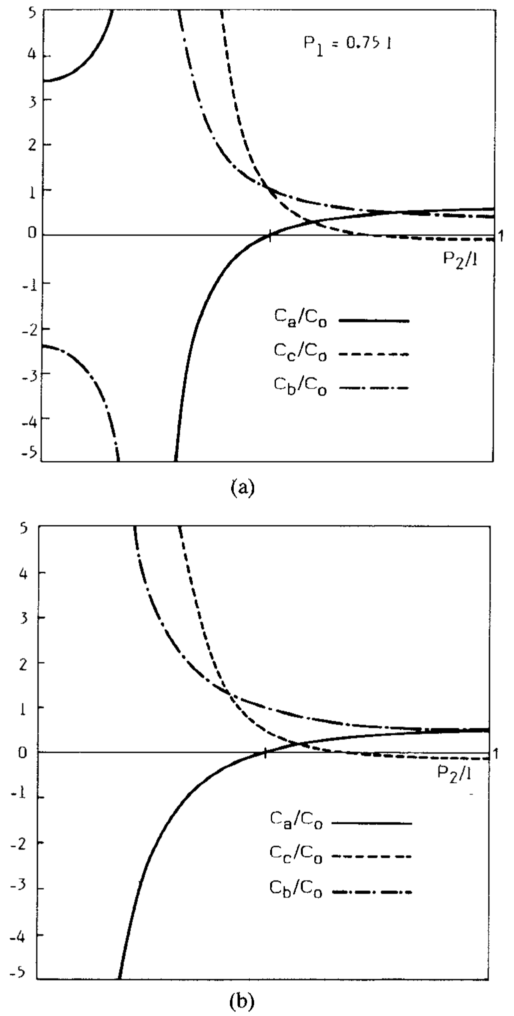

Fig. 3. Behavior of the equivalent circuit capacitors as functions of the position of the second port of the (a) P- and (b) N-elements.

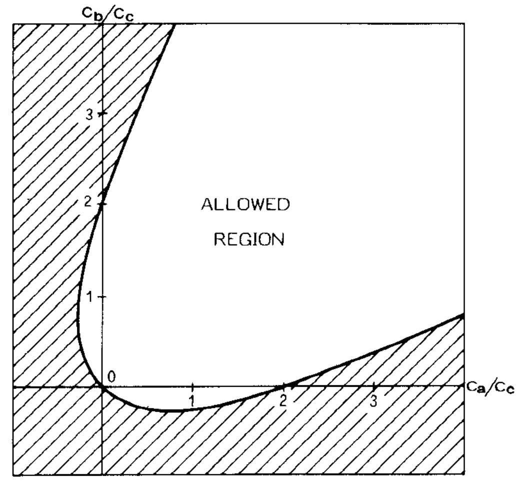

Fig. 4. Allowed region for the capacitance ratios of a P-element in the limit case $w_i \ll l$.

In order to accomplish this step, it is necessary to reverse (4) and (5), i.e., to express the parameters of the microstrip structure in terms of the elements of the equivalent circuit. It is assumed that the feeding lines' widths w_1 and w_2, as well as the substrate thickness t, permeability μ, and relative permittivity ϵ_r are given quantities. Since ϵ, the effective permittivity of the structure, depends on its width b in order to reverse (4) and (5), it is worth introducing as an auxiliary parameter the characteristic impedance Z_0 of a microstrip line of width b; in terms of the equivalent circuit, it is easily found that

$$Z_0 = \frac{\pi}{\omega_1(C_a + C_b)} \tag{7}$$

where

$$w_1 = \left[L_c\left(C_c + \frac{C_a C_b}{C_a + C_b}\right)\right]^{-1/2} \tag{8}$$

is the open-circuit resonant frequency of the equivalent circuit. Once Z_0 has been evaluated, the effective width b and the effective permittivity ϵ can then be calculated using the formulas quoted in [10].

The other geometrical parameters are finally obtained from the following expressions:

$$l = \frac{t(C_a + C_b)}{\epsilon b}$$

$$q_1 = C_b / \sqrt{2(C_aC_b + C_aC_c + C_bC_c)}$$

$$q_2 = -C_a / \sqrt{2(C_aC_b + C_aC_c + C_bC_c)}$$

$$p_1 = \frac{l}{\pi}\cos^{-1}\left[-q_1/\mathrm{Sn}\left(\frac{\pi w_1}{2l}\right)\right]$$

$$p_2 = \frac{l}{\pi}\cos^{-1}\left[q_2/\mathrm{Sn}\left(\frac{\pi w_2}{2l}\right)\right]. \tag{9}$$

The above formulas are valid both for the P-element and, provided (6) is satisfied, for the N-element. The present synthesis procedure, though based on a less approximate model, is much more simple and straightforward than the previous iterative one [6].

The physical realizability of the lumped circuit of Fig. 2 as a rectangular element imposes some constraints on the parameters of the structure; this leads to some conditions on the values of the circuit components, which depend on the width w_1, w_2 of the feeding lines. As an example, the limit case $w_i \ll l$ $(i = 1,2)$, so that $\mathrm{Sn}(\pi w_i/2l)$ may be approximate by unity is considered. In such a case, for a N-element, condition (6) is sufficient to assure the physical realizability of the structure, while for a P-element the following conditions have to be imposed:

$$C_a > \frac{C_b}{2}\,\frac{C_b - 2C_c}{C_b + C_c}$$

$$C_b > \frac{C_a}{2}\,\frac{C_a - 2C_c}{C_a + C_c}. \tag{10}$$

In the case we are interested in, i.e., $C_c > 0$, the above disequations define for C_a/C_c and C_b/C_c the allowed

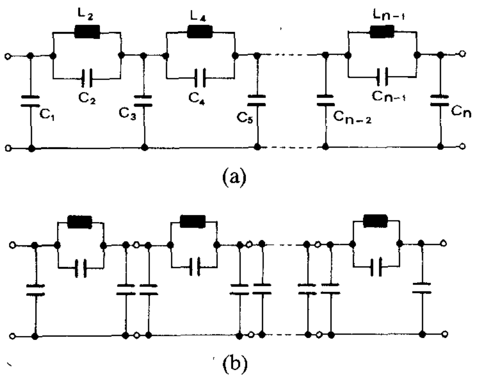

Fig. 5. (a) Elliptic-function low-pass filter prototype and (b) its transformed version suitable for the synthesis.

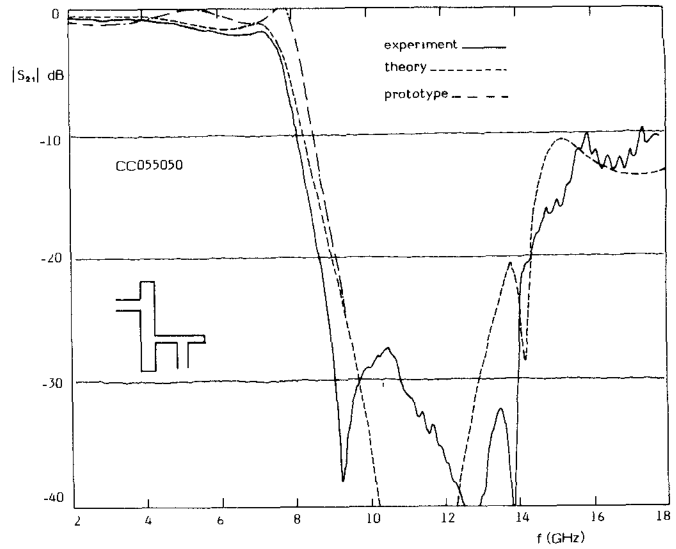

Fig. 6. Theoretical and experimental frequency behavior of the scattering parameter $|S_{21}|$ of the CC.05.50.50 filter fabricated.

region plotted in Fig. 4. It can be seen that a large set of values of capacitance ratios is allowed and, in particular, that symmetrical lumped circuits always satisfy the realizability conditions in the limit case when the widths of the ports are negligible with respect to l.

III. Synthesis of High-Order Elliptic Filters

In the previous section we have stated the possibility of synthesizing third-order elliptic filters as microstrip rectangular elements of the P-type; furthermore, having introduced N-type elements we are now able to realize higher order filters as the cascade connection of both types of microstrip structures.

Fig. 5(a) shows the prototype of a low-pass elliptic filter of nth order (n odd). This network can be transformed into the scheme of Fig. 5(b) constituted by the cascade of m elementary cells $[m = (n-1)/2]$ in such a way that each cell corresponds to a single rectangular element. Since the nth order filter of Fig. 5(a) contains $(3n-1)/2$ reactive elements and the P- and N-elements possess four and three degree of freedom, respectively, it can be easily seen that the synthesis of such a filter requires the use of $(m-1)$ N-elements and one P-element. The position of the latter in the cascade is arbitrary.

Supposing the substrate characteristics (ϵ_r, μ, t) and the feeding line impedances are given, the synthesis procedure is straightforward and consists of the following steps:

1) Choose the position k of the P-elements in the cascade, k being an integer between 1 and m.

2) Identify the first $k-1$ and the last $m-k$ cells of the circuit of Fig. 6(b) using relation (6) iteratively.

3) Synthesize each planar element from its equivalent cell following the procedure described in the previous section; in such a way all the geometrical parameters of the entire structure are obtained.

Contrary to other nonredundant synthesis techniques of microstrip filters which are based on the translation of each lumped element of the prototype into a correspondent line section, in the present technique each microstrip element corresponds to four reactive elements of the prototype. This reduces the parasitics and unwanted reactances, which compromise in the classical approach the filter performance; moreover, the present method makes it possible to realize high-order low-pass filters with size reduction, lower pass-band attenuation characteristics with respect to similar structures designed with redundant synthesis methods.

IV. Experimental Results

Following the procedure described, a number of low-pass elliptic filters have been fabricated using a 0.635-mm thick alumina substrate ($\epsilon_r = 10$).

Fig. 6 shows the experimental behavior of the scattering parameters $|S_{21}|$ of the fifth-order CC.05.50.50 filter in the frequency band 2–18 GHz. The filter has been designed with a cutoff frequency of 8 GHz and consists of the cascade of a P-element and a N-element, whose dimensions are

P-element:	$l = 7.46$ mm $b = 0.98$ mm $p_1 = 5.67$ mm $w_1 = 0.6$ mm $p_2 = 5.30$ mm $w_2 = 0.29$ mm
N-element:	$l = 5.27$ mm $b = 0.29$ mm $w_1 = 0.29$ mm $p_2 = 2.71$ mm $w_2 = 0.6$ mm.

For comparison, the prototype behavior and the theoretical behavior based on the multimode analysis [7] are plotted in the same figure; differences between them are only due to the higher order modes which are not taken into account in the synthesis. On the contrary, differences between theory and experiment are due to other phenomena, such as ohmic and radiation loss, excitation of surface waves, mismatching of the launchers, which have been neglected in the theoretical analysis; moreover, a unique

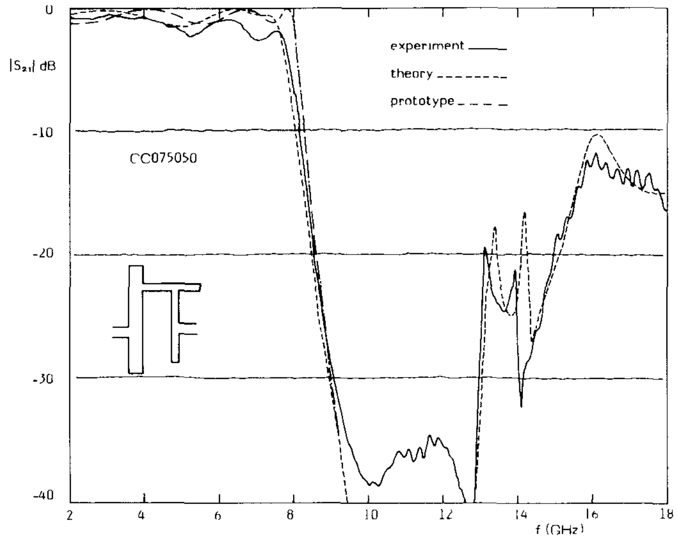

Fig. 7. Theoretical and experimental frequency behavior of the scattering parameter $|S_{21}|$ of the CC.07.50.50 filter fabricated.

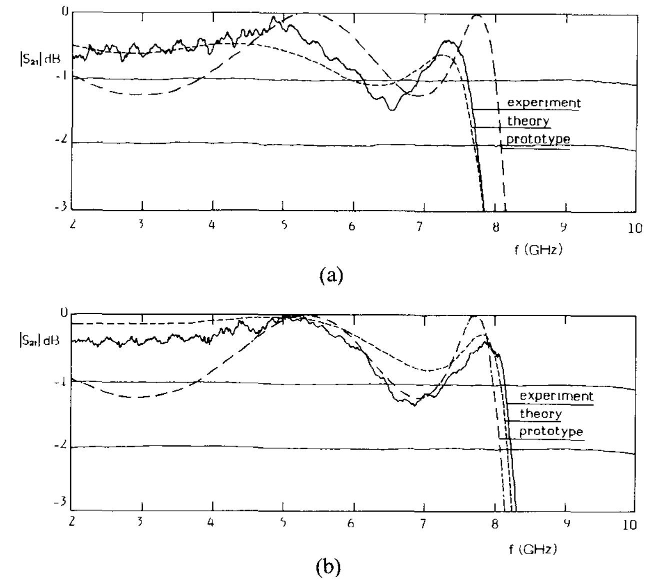

Fig. 8. Theoretical and experimental passband behavior of $|S_{21}|$ of the CC.05.50.50 filter of (a) Fig. 6 and of (b) the adjusted version.

effective permittivity has been used for all the resonant modes.

The results obtained for a seventh-order filter are plotted in Fig. 7. This is a CC.07.50.50 designed with the same cutoff frequency of 8 GHz and consisting of the cascade of one P- and two N-elements whose dimensions are

P-element: $l = 7.52$ mm $b = 1.01$ mm $p_1 = 1.76$ mm $w_1 = 0.6$ mm $p_2 = 1.99$ mm $w_2 = 0.40$ mm
N-element: $l = 5.75$ mm $b = 0.40$ mm $w_1 = 0.40$ mm $p_2 = 3.10$ mm $w_2 = 0.48$ mm
N-element: $l = 5.58$ mm $b = 0.48$ mm $w_1 = 0.48$ mm $p_2 = 3.21$ mm $w_2 = 0.6$ mm.

The performance of the filters can be considered highly satisfactory up to about 13 GHz; furthermore, the measurement of the delay time of the CC.07.50.50 filter resulted in 0.096 ns in front of the theoretical value of 0.094 ns of the prototype. The presence of a pseudopassband, due to high-order resonant modes, constitutes the main limitation of the present synthesis technique. In order to provide a substained stopband attenuation though preserving a sharp selectivity, a conventional low-pass section with a higher cutoff frequency could be added to the elliptic filter [12]. On the other hand, the passband behavior can be further improved by a simple adjusting procedure which consists of the following steps:

1) Evaluate theoretically the contribution of higher order modes to the Z-matrix of the filter synthesized, at a given frequency in the passband;

2) Subtract such a contribution to the Z-matrix of the prototype, in order to obtain a new filter structure by the same synthesis procedure, described in the previous section.

The results of such an adjusting procedure are shown in Fig. 8(a) and (b), where the passband behavior of the scattering parameters $|S_{21}|$ of the above CC.05.50.50 filter and of the adjusted version are plotted. The adjusting procedure, performed at 7.5 GHz, results in a clear improvement of the passband behavior of the filter. In particular, a lower attenuation than both the prototype and the first version of the filter was achieved below 5 GHz.

V. Conclusions

A new method for synthesizing nonredundant low-pass elliptic filters in microstrip configuration has been presented. The synthesis is based on an equivalence between a proper rectangular microstrip structure and an elementary cell of the lumped prototype. The advantages obtained are size reduction and low insertion loss with respect to redundant synthesis techniques and an effective control of parasitics and unwanted reactances with respect to syntheses based on quasi-lumped elements. Experimental results of fifth- and seventh-order filters show an excellent passband behavior; the problem of the limited width of the stopband can be overcome, while preserving the high selectivity and low insertion loss in the passband, by adding a conventional low-pass section with higher cutoff frequency [11], [12].

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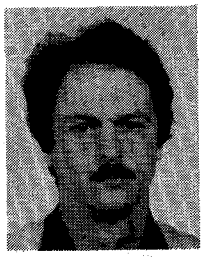

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